MULTI-OBJECTIVE OPTIMIZATION FRAMEWORK FOR FINITE ELEMENT MODEL UPDATING AND RESPONSE PREDICTION VARIABILITY

E. Ntotsios and C. Papadimitriou
University of Thessaly, Dep. of Mechanical and Industrial Engineering, Volos, Greece
entotsio@uth.gr, costasp@uth.gr

Abstract

A multi-objective optimization framework based on modal data is presented for finite element model updating in structural dynamics. The framework results in multiple Pareto optimal structural models that are consistent with the measured data and the norms used for reconciling finite element models with data. Computationally efficient methods for estimating the gradients and Hessians of the objective functions with respect to the model parameters are proposed and shown to significantly reduce the computational effort for solving the single or multi-objective optimization problems. Theoretical and computational developments are addressed and demonstrated by updating the finite element model of a concrete bridge structure using modal data identified from ambient acceleration time history measurements. The results clearly indicate that there is wide variety of Pareto optimal structural models that trade off the fit in various measured quantities. The variability in Pareto models affect the variability in response predictions.

Introduction

Structural model updating methods have been proposed in the past to reconcile mathematical models, usually finite element models, with experimental data. The estimate of the optimal model from a class of models is sensitive to uncertainties that are due to limitations of the mathematical models used to represent the behavior of the real structure, the presence of measurement error in the data, the number and type of measured data used in the reconciling process, as well as the norms used to measure the fit between measured and model predicted characteristics. The optimal structural models resulting from such methods can be used for improving the model response and reliability predictions (Papadimitriou et al. 2001).

Structural model parameter estimation problems based on measured data, such as modal characteristics (e.g. Fritzen et al. 1998; Teughels and De Roeck 2005) or response time histories (Beck and Katafygiotis 1998), are often formulated as weighted least-squares problems in which metrics, measuring the residuals between measured and model predicted characteristics, are build up into a single weighted residuals metric formed as a weighted average of the multiple individual metrics using weighting factors. Standard optimization techniques are then used to find the optimal values of the structural parameters that minimize the single weighted residuals metric representing an overall measure of fit between measured and model predicted characteristics. Due to model error and measurement noise, the results of the optimization are affected by the values assumed for the weighting factors. The model updating problem has also been formulated in a multi-objective context (Christodoulou and Papadimitriou 2007) that allows the simultaneous minimization of the multiple metrics, eliminating the need for using arbitrary weighting factors for weighting the relative importance of each metric in the overall measure of fit. The multi-objective parameter estimation methodology provides multiple Pareto optimal structural models consistent with the data and the residuals used.

In this work, the structural model updating problem using modal residuals is formulated as a multi-objective optimization problem and as a single-objective optimization with the objective formed as a weighted average of the multiple objectives using weighting factors. Theoretical and computational issues arising in multi-objective identification are addressed and the correspondence between the multi-objective identification and the weighted residuals identification is established. In addition, computational issues associated with solving the resulting multi-objective and single-objective optimization problems are addressed. Computationally efficient algorithms for estimating the gradients and Hessians of the objective functions are proposed and shown to significantly reduce the computational effort and the number of iterations required for convergence.

Model Updating Based on Modal Residuals

Let $D=\{\hat{\omega}_r, \underline{\hat{\phi}}_r \in R^{N_0},\ r=1,\cdots,m\}$ be the measured modal data from a structure, consisting of modal frequencies $\hat{\omega}_r$ and modeshape components $\underline{\hat{\phi}}_r \in R^{N_0}$ at N_0 measured DOFs, where m is the number of observed modes. Consider a parameterized class of linear structural models used to model the dynamic behavior of the structure and let $\underline{\theta} \in R^{N_\theta}$ be the set of free structural model parameters to be identified using the measured modal data. The objective in a modal-based structural identification methodology is to estimate the values of the parameter set $\underline{\theta}$ so that the modal data $\{\omega_r(\underline{\theta}),\ \underline{\phi}_r(\underline{\theta}) \in R^{N_d}, r=1,\cdots,m\}$, where N_d is the number of model degrees of freedom (DOF), predicted by the linear class of models best matches, in some sense, the experimentally obtained modal data in D. For this, let

$$\varepsilon_{\omega_r}=\varepsilon(\omega_r,\hat{\omega}_r)=\frac{\omega_r^2-\hat{\omega}_r^2}{\hat{\omega}_r^2} \qquad \text{and} \qquad \varepsilon_{\underline{\phi}_r}=e(L\underline{\phi}_r,\underline{\hat{\phi}}_r)=\frac{\left\|\beta_r L\underline{\phi}_r-\underline{\hat{\phi}}_r\right\|}{\left\|\underline{\hat{\phi}}_r\right\|} \tag{1}$$

$r=1,\cdots,m$, be the measures of fit or residuals between the measured modal data and the model predicted modal data for the r-th modal frequency and modeshape components, respectively, where $\|\underline{z}\|^2=\underline{z}^{\mathrm{T}}\underline{z}$ is the usual Euclidian norm, and $\beta_r=\underline{\hat{\phi}}_r^T L\underline{\phi}_r / \left\|L\underline{\phi}_r\right\|^2$ is a normalization constant that guaranties that the measured modeshape $\underline{\hat{\phi}}_r$ at the measured DOFs is closest to the model modeshape $\beta_r L\underline{\phi}_r(\underline{\theta})$ predicted by the particular value of $\underline{\theta}$. The matrix $L \in R^{N_0 \times N_d}$ is an observation matrix comprised of zeros and ones that maps the N_d model DOFs to the N_0 observed DOFs.

The measured modal properties are grouped into n groups. Each group contains one or more modal properties. The modal properties assigned in the i-th group are identified by the set $g_i(k)$, $i=1,\cdots,n$ and $k=1,2$, with any element in the set $g_i(k)$ is an integer from 1 to m. The elements in the set $g_i(k)$ with $k=1$ refer to the number of the measured modal frequency assigned in the group i, while the elements of the set $g_i(k)$ with $k=2$ refer to the number of the measured modeshape assigned in the group i. For the ith group, a norm $J_i(\underline{\theta})$ is introduced to measure the residuals of the difference between the measured values of the modal properties involved in the group and the corresponding modal values predicted from the model class for a particular value of the parameter set $\underline{\theta}$. The measure of fit in a modal group is the sum of the individual square errors in (1) for the corresponding modal properties involved in the modal group. Specifically, the measure of fit is given by

$$J_i(\underline{\theta}) = \sum_{r \in g_i(1)} \varepsilon^2(\omega_r, \hat{\omega}_r) + \sum_{r \in g_i(2)} e^2(L\underline{\phi}_r, \hat{\underline{\phi}}_r) \tag{2}$$

The grouping of the modal properties $\{\omega_r(\underline{\theta}),\ \underline{\phi}_r(\underline{\theta}), r = 1, \cdots, m\}$ into n groups and the selection of the measures of fit (residuals) $J_1(\underline{\theta}), \cdots, J_n(\underline{\theta})$ are usually based on user preference. The modal properties assigned to each group are selected by the user according to their type and the purpose of the analysis. The aforementioned analysis accommodates general grouping schemes and objective functions. A specific grouping scheme is to assume two groups, $n = 2$. The first group contains only modal frequencies and the second group contains only modeshapes, i.e. $g_1(1) = \{1, \ldots, n\}$, $g_2(2) = \{1, \ldots, n\}$ while $g_1(2)$ and $g_2(1)$ are empty sets.

Multi-Objective Identification

The problem of identifying the model parameter values that minimize the modal or response time history residuals can be formulated as a multi-objective optimization problem stated as follows (Haralampidis et al. 2005). Find the values of the structural parameter set $\underline{\theta}$ that simultaneously minimizes the objectives

$$\underline{y} = \underline{J}(\underline{\theta}) = (J_1(\underline{\theta}), \cdots, J_n(\underline{\theta})) \tag{3}$$

For conflicting objectives $J_1(\underline{\theta}), \cdots, J_n(\underline{\theta})$, there is no single optimal solution, but rather a set of alternative solutions, known as Pareto optimal solutions, that are optimal in the sense that no other solutions in the parameter space are superior to them when all objectives are considered. The multiple Pareto optimal solutions are due to modeling and measurement errors.

Weighted Modal Residuals Identification

The parameter estimation problem is also solved by minimizing the single objective

$$J(\underline{\theta}; \underline{w}) = \sum_{i=1}^{n} w_i J_i(\underline{\theta}) \tag{4}$$

formed from the multiple objectives $J_i(\underline{\theta})$ using the weighting factors $w_i \geq 0$, $i = 1, \cdots, n$, with $\sum_{i=1}^{n} w_i = 1$. The objective function $J(\underline{\theta}; \underline{w})$ represents an overall measure of fit between the measured and the model predicted characteristics. The relative importance of the residual errors in the selection of the optimal model is reflected in the choice of the weights. The results of the identification depend on the weight values used. It can be readily shown that the optimal solution to the problem is one of the Pareto optimal solutions. Conventional weighted least squares methods assume equal weight values, $w_1 = \cdots = w_n = 1/n$. This conventional method is referred herein as the equally weighted method.

Computational Issues Related to Model Updating Formulations

The proposed single and multi-objective identification problems are solved using available single and multi objective optimization algorithms. The optimization of $J(\underline{\theta};\underline{w})$ in (4) with respect to $\underline{\theta}$ for given $\underline{w}$ can readily be carried out numerically using any available algorithm for optimizing a nonlinear function of several variables. These single objective optimization problems may involve multiple local/global optima. A hybrid optimization algorithm that exploits the advantages of Evolution Strategies (ES) and gradient-based methods has been developed to detect the neighborhood of the global optimum and then using gradient information to accelerate convergence to the global optimum (Christodoulou and Papadimitriou 2007).

The set of Pareto optimal solutions can be obtained using available multi-objective optimization algorithms. The Normal-Boundary Intersection (NBI) method (Das and Dennis 1998) is a very efficient algorithm for solving the multi-objective optimization problem. It produces an evenly spread of points along the Pareto front, even for problems for which the relative scaling of the objectives are vastly different. The NBI optimization method involves the solution of constrained nonlinear optimization problems using available gradient-based constrained optimization methods. The NBI uses the gradient information to accelerate convergence to the Pareto front.

Gradient and Hessian Computations

In order to guarantee the convergence of the gradient-based optimization methods for structural models involving a large number of DOFs with several contributing modes, the gradient of the objective function with respect to the parameter set $\underline{\theta}$ has to be estimated accurately. It has been observed that numerical algorithms such as finite difference methods for gradient evaluation does not guarantee convergence Moreover, gradient computations with respect to the parameter set using the finite difference method requires the solution of as many eigenvalue problems as the number of parameters.

Analytical expressions for the gradient of the modal frequencies and modeshapes can be used to overcome the convergence problems. In particular, Nelson's method (Nelson 1978) is used for computing analytically the first derivatives of the eigenvalues and the eigenvectors. The advantage of the Nelson's method compared to other methods is that the gradient of eigenvalue and the eigenvector of one mode are computed from the eigenvalue and the eigenvector of the same mode and there is no need to know the eigenvalues and the eigenvectors from other modes. For each parameter in the set $\underline{\theta}$ this computation is performed by solving a linear system of the same size as the original system mass and stiffness matrices. Nelson's method has also been extended in this work to compute the second derivatives of the eigenvalues and the eigenvectors.

Finally, the computation of the gradients and the Hessian of the objective functions is shown to involve the solution of a single linear system, instead of N_θ linear systems required in usual computations of the gradient and $N_\theta\left(N_\theta+1\right)$ linear systems required in the computation of the Hessian. This reduces considerably the computational time, especially as the number of parameters in the set $\underline{\theta}$ increase. The expressions for the first and second derivatives of the objective functions are next presented. Due to space limitations details of the deviations are not shown.

The gradient of square errors $\varepsilon^2_{\omega_r}(\underline{\theta})$ and $\varepsilon^2_{\phi_r}(\underline{\theta})$ involved in objectives (2) are given by

$$\frac{\partial \varepsilon^2_{\omega_r}(\underline{\theta})}{\partial \theta_j} = \left[\frac{2\varepsilon_{\omega_r}(\underline{\theta})}{\hat{\omega}_r^2}\underline{\phi}_r^T\right](K_j - \omega_r^2 M_j)\underline{\phi}_r \tag{5}$$

and

$$\frac{\partial \varepsilon^2_{\phi_r}(\underline{\theta})}{\partial \theta_j} = \underline{x}_r^{*T}\underline{F}_{r,j} \tag{6}$$

where $\underline{F}_{r,j} = -(I - M\underline{\phi}_r\underline{\phi}_r^T)(K_j - \omega_r^2 M_j)\underline{\phi}_r$ and $\underline{x}_r$ is given by the solution of the linear system

$$A_r^* X_r = D_r \tag{7}$$

with $D_r = L^T 2\beta_r\left(\beta_r L\underline{\phi}_r - \underline{\hat{\phi}}_r\right)/\left\|\underline{\hat{\phi}}\right\|^2$ and X_r replaced by $\underline{x}_r$. For notational convenience, the dependence of several variables on the parameter set $\underline{\theta}$ has been dropped. For an $n\times n$ matrix $A_r = K - \omega_r^2 M$, A_r^* is used to denote the modified matrix derived from A_r by replacing the elements of the k-th column and the k-th row by zeroes and the (k,k) element of A_r by one, where k denotes the element of the modeshape vector $\underline{\phi}_r$ with the highest absolute value. Also, the n vector $\underline{b}_r^*$ is used to denote the modified vector derived from $\underline{b}_r$ by replacing the k-th element of the vector $\underline{b}_r$ by zero. Also, K_j and M_j in the formulation denote the quantities $\partial K/\partial\theta_j$ and $\partial M/\partial\theta_j$ that can be obtained either analytically or numerically using finite element methods.

Similarly, it can be shown that the (i,j) element of the Hessian of $\varepsilon^2_{\underline{\omega}_r}(\underline{\theta})$ and $\varepsilon^2_{\underline{\varphi}_r}(\underline{\theta})$ can be adequately approximated in the form (assuming that $M_j = 0$)

$$\frac{\partial^2 \varepsilon^2_{\omega_r}(\underline{\theta})}{\partial\theta_i\partial\theta_j} = \frac{2}{\hat{\omega}_r^4}[\underline{\phi}_r^T(K_i - \omega_r^2 M_i)\underline{\phi}_r][\underline{\phi}_r^T(K_j - \omega_r^2 M_j)\underline{\phi}_r] \tag{8}$$

and

$$\frac{\partial^2 \varepsilon^2_{\varphi_r}(\underline{\theta})}{\partial\theta_i\partial\theta_j} = -\frac{2}{\left\|\underline{\hat{\phi}}\right\|^2\left\|L\underline{\phi}_r\right\|^2}\left(\underline{z}_r^{*T}\underline{F}_{r,i}\right)\left(\underline{z}_r^{*T}\underline{F}_{r,j}\right) - \beta_r^2\left\|L\underline{\phi}_r\right\|^2\underline{F}_{r,j}^{*T}X_r X_r^T\underline{F}_{r,i}^* \tag{9}$$

where $\underline{z}_r$ is given by the solution of the linear system (7) with $D_r = \left(I - M^T\underline{\phi}_r\underline{\phi}_r^T\right)L^T\left(2\beta_r L\underline{\phi}_r - \underline{\hat{\phi}}_r\right)$ and X_r is given by (7) with $D_r = \left(I - M^T\underline{\phi}_r\underline{\phi}_r^T\right)L^T$.

It is clear that the computation of the first and second derivatives of the square errors for the modal properties of the r-th mode with respect to the parameters in $\underline{\theta}$ requires only the solutions of the linear system (7), independent of the number of parameters in $\underline{\theta}$. For a large number of parameters in the set $\underline{\theta}$, the above formulation for the gradients and

Hessian of the mean errors in modal frequencies and in the modeshape components in (1) are computationally very efficient and informative.

Application

The proposed framework is applied to a R/C bridge (Figure 1a) of Egnatia Odos motorway. The response to ambient excitation caused by traffic and wind has been systematically monitored using an array of 24 accelerometers. Available modal identification methods are used to identify the modes by processing the ambient vibrations. To implement the model updating techniques, an appropriate parametric finite element model of the bridge is considered using three-dimensional two-node beam-type finite elements to model the deck, the piers and the bearings. This model is shown in Figure 1b and has 1038 degrees of freedom. A three parameter model class is employed with θ_1 accounting for the stiffness of the elastomeric bearings at the abutments, θ_2 accounting for the stiffness of the deck, and θ_3 accounting for the stiffness of the piers. The model class is updated using the three modal frequencies and modeshapes and two modal groups, the first containing the modal frequencies and the second one the modeshapes.

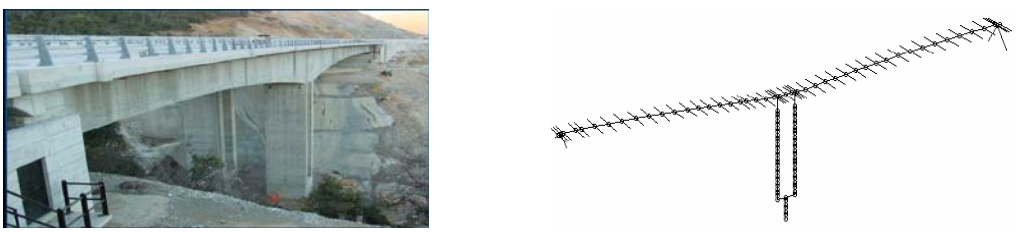

Figure 1. (a) View of the Polymilos bridge, (b) Finite element model.

The results from the multi-objective identification methodology are shown in Figure 2, along with the single solution obtained using the equally weighted method (EWM). For each model class and associated structural configuration, the Pareto front, giving the Pareto solutions in the two-dimensional objective space, is shown in Figure 2a. The non-zero size of the Pareto front and the non-zero distance of the Pareto front from the origin are due to modeling and measurement errors. Specifically, the distance of the Pareto points along the Pareto front from the origin is an indication of the size of the overall measurement and modeling error. The size of the Pareto front depends on the size of the model error and the sensitivity of the modal properties to the parameter values $\underline{\theta}$ (Christodoulou and Papadimitriou 2007). Figure 2b show the corresponding Pareto optimal solutions in the two-dimensional parameter space (θ_1, θ_2). It is observed that a wide variety of Pareto optimal solutions are obtained for different structural configurations that are consistent with the measured data and the objective functions used. The Pareto optimal solutions are concentrated along a one-dimensional manifold in the three-dimensional parameter space. All Pareto solutions correspond to acceptable compromise structural models trading-off the fit in the modal frequencies involved in the first modal group with the fit in the modeshape components involved in the second modal groups. The identified variability in Pareto optimal solutions has demonstrated by

Christodoulou and Papadimitriou (2007) to considerably affect the variability in the response predictions.

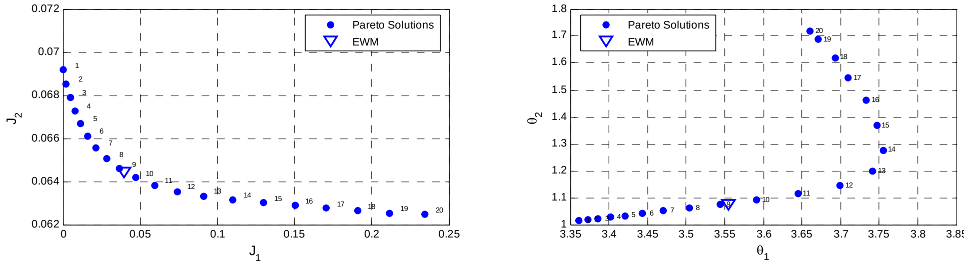

Figure 2. Pareto front and Pareto optimal solutions in the (a) objective space and (b) parameter space (θ_1, θ_2)

Conclusions

Computationally efficient model updating algorithms were developed to compute all Pareto optimal structural models consistent with the measured data and the norms used to measure the fit between the measured and model predicted modal properties. Application on a R/C bridge demonstrated that a wide variety of Pareto optimal structural models consistent with the measured modal data can be obtained. The variability in the Pareto optimal models is due to the model and measurement error. The large variability in the Pareto optimal models results in large variability in the response and structural reliability predictions.

Acknowledgements

This research was co-funded 75% from the European Union (European Social Fund), 25% from the Greek Ministry of Development (General Secretariat of Research and Technology) and from the private sector, in the context of measure 8.3 of the Operational Program Competitiveness (3rd Community Support Framework Program) under grant 03-EΔ-524 (PENED 2003). This support is gratefully acknowledged.

References

Beck, J.L. and L.S. Katafygiotis (1998), "Updating models and their uncertainties- I: Bayesian statistical framework," *Journal of Engineering Mechanics (ASCE)*, **124**, 455-461.

Christodoulou, K. and C. Papadimitriou (2007), "Structural Identification Based on Optimally Weighted Modal Residuals," *Mechanical Systems and Signal Processing*, **21**, 4-23.

Das, I. and J.E. Jr. Dennis (1998), "Normal-Boundary Intersection: A new method for generating the Pareto surface in nonlinear multi-criteria optimization problems," *SIAM Journal of Optimization*, **8**, 631-657.

Fritzen, C.P., D. Jennewein, and T. Kiefer (1998), "Damage detection based on model updating methods," *Mechanical Systems and Signal Processing*, **12**(1), 163-186.

Nelson, R.B. (1976), "Simplified calculation of eigenvector derivatives", *AIAA Journal* **14**(9), 1201-1205.

Papadimitriou, C., J.L. Beck, and L.S. Katafygiotis (2001), "Updating robust reliability using structural test data," *Probabilistic Engineering Mechanics*, **16**, 103-113.

Teughels, A. and G. De Roeck (2005), "Damage detection and parameter identification by finite element model updating," *Archives of Computational Methods in Engineering*, **12**(2), 123-164.